

**An Exploration of Decision Making and Game Theory  
With Applications to Economic Situations**

An Honors Thesis (Honors 499)

by

Jeffrey W. Cline

Thesis Advisor  
Lee C. Spector

A handwritten signature in black ink, appearing to read "Lee C. Spector". The signature is fluid and cursive, with a long horizontal stroke extending from the end.

Ball State University  
Muncie, Indiana

April 2012

Expected Date of Graduation  
May 2012

510 11  
2121  
4234  
15  
2429  
1211  
1111  
1011

### **Abstract:**

From the moment we wake up in the morning to the moment we go to sleep at night, we are almost constantly making decisions. From the simple, such as brushing our teeth before bed, to the complex, for example planning our retirement, decisions play an important role in our daily lives. Despite this, it is not very often that we take time to think about how we go about making our decisions in order to achieve the best possible outcomes. This paper explores theories which attempt to describe the ways in which we make decisions. The first two sections explore economic decisions made at the individual level. The next two sections explore decision making in a more macroeconomic context. Finally, the paper ends with a section that introduces certain aspects of game theory and explores two macroeconomic games.

### **Acknowledgements:**

I would like to thank Dr. Lee Spector for his input and guidance throughout the duration of this project. His help during the process is only a small portion of all that he has done to encourage and support me during my four years at Ball State. Without his direction, I would not have developed into the student I am today.

**Table of Contents:**

Introduction.....1

1. Cases Where Optimal Decisions Depend upon the Actions of Others.....2

2. An Introduction and Exploration of the Permanent Income Hypothesis.....5

3. An Introduction of Uncertainty and Application to Tax Policy.....14

4. An Introduction to Time Inconsistency and Application to Discretionary Policy.....20

5. An Introduction of Game Theory and Two Macroeconomic Games.....25

Conclusion.....34

Works Cited.....35

Appendix: Another Application of the Permanent Income Hypothesis.....36

## **Introduction:**

Consider the situation we all face the moment we wake up in the morning. The alarm clock is going off, the room is dark, and the bed is warm and inviting. The first thing that we do at this moment is decide whether to shut off the alarm clock or to hit the snooze button and allow ourselves another five minutes in our cozy beds. Sometimes this situation is repeated several times before we actually decide to shut off the alarm, climb out of bed, and begin our day. In this manner, we all begin by making a decision the moment we wake up in the morning, albeit a very simple decision. If we take a moment to consider our typical day, we can conclude that our lives are filled with situations in which we must make decisions. Some of these decisions are relatively simple, such as our initial decision to get up in the morning. Other decisions we make can be extremely complex, for example when we meet with a financial adviser to establish a long term plan of saving money for retirement. Regardless of the complexity level, the decisions we make are most often aimed at achieving an optimal outcome given the conditions we face.

Despite this pervasiveness of decision making in our lives, it is not often that we take time to think about the processes we use to make these decisions. In this paper, we will endeavor to accomplish this task by exploring several decision making conditions, both at an individual level and at an institutional level. The paper will be organized to flow generally from less complex ideas into more complicated ideas, exploring each in detail and discussing any implications that result from those ideas. The first two sections develop the decision making process and explore applications at the level of individual people or firms. The third and fourth sections shift focus toward big picture, institutional decision making. The final section introduces concepts from game theory and their application to institutional decision making.



## **1. Cases Where Optimal Decisions Depend upon the Actions of Others:**

It is often the case that the decisions people make are not as simple as choosing between two or more actions where the outcome of each action is known. In fact, many decisions that are made every day follow the form of one person or institution (hereafter referred to as an agent) trying to make an optimal choice. Agents do this by establishing one or more criteria, but the optimal choice is often dependent on the actions or decisions of another agent. In such situations, two important factors must be considered by an agent. First, the agent must establish criteria to distinguish between choices. Second, the actions of other agents must be considered within these criteria. Once these two factors have been considered, the task of decision making becomes a process of comparing choices. The comparison is based on both the established criteria and the other agent's actions. The comparison is followed by selecting the choice which yields the optimal outcome. This decision making process can transition from a relatively simple to a rather complex exercise when the actions of another agent must also be considered. To demonstrate this phenomenon, two examples will be examined. Both examples will begin with a relatively simple decision based only on one criterion, followed by a transition to a more complex process based on the criterion and the actions of another agent.

Suppose that John and Fred are playing tennis. John has served the ball and Fred has made a return hit which John is now preparing to strike. The players are positioned as shown in Figure 1.1 (J & F<sub>1</sub>). Based on the current situation, John knows he can hit a baseline shot (represented by  $S_1$  in Figure 1.1) or a drop shot ( $S_2$  in Figure 1.1). John's criterion for shot selection is such that he wants to hit the ball to where Fred has the lowest probability of being able to make a return shot. If Fred has a 50% chance of returning the ball from  $S_1$  and a 30%

chance of returning the ball from  $S_2$ , then John's selection criterion indicates he should choose  $S_2$  for the shot. At this point, it should be noted that John's decision is concerned only with the selection criterion and does not consider the actions Fred may take before John strikes the ball. Let us now add Fred's actions to the decision making process.

To add this second dimension to the situation, let us start by assuming that Fred has two actions that he may take before John hits the ball. He may either remain on the baseline in position  $F_1$ , or he may rush the net, moving to position  $F_2$ . John must now alter his decision making process to include Fred's actions. As before, if Fred is at  $F_1$ , he has a 50% chance of returning the baseline shot  $S_1$  and a 30% chance of returning the drop shot  $S_2$ , so John should hit the drop shot. However, if Fred chooses to move to position  $F_2$ , he then has a 60% chance of returning John's shot to  $S_1$  and a 75% chance of returning John's shot to  $S_2$ . Based on Fred's action, John should now choose the baseline shot at  $S_1$  over the drop shot at  $S_2$  in order to minimize Fred's chance of returning the shot.

**Figure 1.1**

	$S_1$	$F_1$	
	$S_2$	$F_2$	
	J		

Let us now consider a more economic example involving the strategies of competing firms. Suppose a rural region of a country has experienced considerable development and

expansion over the past several years. As a result, demand for electricity has increased beyond the capacity of the current power plant in the region which is owned by Power Supply, Inc. (PSI). Big Electric Corporation (BEC) has taken note of the opportunity and has initiated the construction of a new power plant in the area. BEC may construct either a large plant  $L$  or a small plant  $S$ . BEC's criterion for size selection is the expected profitability of the new plant, with higher expected profitability corresponding to a more optimal decision. Table 1.1 displays the expected profitability (in millions of dollars) for each company, depending on BEC's choice of plant size ( $S$  or  $L$ ). The first numbers of the right hand column in Table 1.1 correspond to BEC's profitability, the second to that of PSI. For example, if BEC chooses  $S$ , both BEC and PSI can expect a profit of \$5 million each. Conversely, if BEC chooses  $L$ , it can expect a profit of \$8 million while PSI's expected profit is only \$2 million. Based on this information, BEC's optimal decision is to construct a large plant.

To complicate the situation, let us assume that PSI has also taken notice of the increased demand for electricity. PSI does not want to build an additional facility in the region, but it does have the option of expanding its current small plant to a large plant before BEC beings construction. This complicates BEC's decision, because if both companies choose to operate large power plants in the region, supply of electricity will increase to the point that there will be considerable downward pressure on electricity prices. The reduction in price will in turn make the large power plants unprofitable in the region. This situation is modeled in Table 1.2. The left hand column represents BEC's choice to build a small  $S$  or large  $L$  plant. The top row represents PSI's choice to not expand  $N$  or expand  $E$ . Again, the first numbers in Table 1.2 relate to BEC's expected profits, the second numbers to PSI. It should be clear that BEC must carefully consider

PSI's decision of whether or not the current plant will be expanded. This is because if PSI plays conservatively and decides not to expand  $N$ , then it is in BEC's best interest to build a large plant and again expect \$8 million in profit while PSI expects only \$2 million. However, if PSI is aggressive and does expand  $E$ , then the optimal choice for BEC is to build a small plant. This is the case because building a large plant results in an expected loss of \$5 million, while building a small plant results in an expected profit of \$2 million for BEC.

**Table 1.1**

BEC	PSI
$S$	5 5
$L$	8 2

**Table 1.2**

BEC ↓ PSI →	$N$	$E$
$S$	5 5	2 8
$L$	8 2	-5 -5

## **2. An Introduction and Exploration of the Permanent Income Hypothesis:**

While understanding the decision making framework of competing tennis players or electricity firms may be helpful given certain situations, these decisions are by nature relatively small in scale. Let us now expand our scope to gain an understanding of the decision making framework that relates to factors which have a significant impact on entire economies. These decision making frameworks are arguably more helpful. One such factor that can have a large magnitude effect on economies is consumption. Unfortunately, the concepts that underlie consumption decisions are more complex and less intuitive than those already discussed. To exacerbate this problem, there have been multiple theories regarding consumption developed throughout history. For the sake of time and clarity, just one of these theories will be discussed. This particular theory was set forth by Modigliani and Brumberg, and further developed by Friedman. The theory is called the Permanent Income Hypothesis. Let us first momentarily

depart from the study of decision making to develop the ideas in the Permanent Income Hypothesis. Once this has been accomplished, some examples can be explored which demonstrate the influence the theory has on optimal consumption decisions.

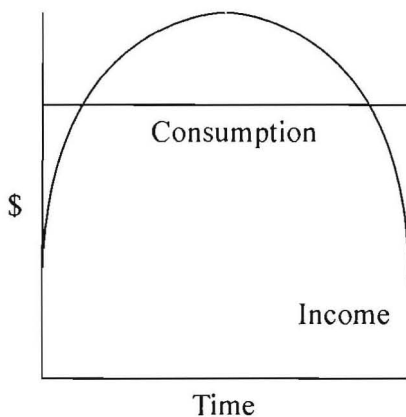
The idea behind Permanent Income Hypothesis is that consumption is a far more complex decision than simply basing current consumption on current income. The theory argues that agents base their consumption over a much longer income time horizon. Consider a recently married couple who are buying a house. It is unlikely that they have the means to afford this consumption based entirely on current income. They are likely to use past income (in the form of savings), current income, and future income (in the form of loans) to finance their consumption. This single act of consumption is a highly complex decision based on a long time horizon which includes a mix of past income saved, present income earned, and future income foregone.

The idea of planning consumption over a long period of time can be expanded to include agents in all stages of life. It is able explain why college students are willing to accumulate so much debt, why people in the workforce often spend less than they earn (i.e. they save), and why people retire and live off of what they have saved in their working years. This idea begs for a conceptual framework that can describe how agents make decisions regarding consumption based not only on their current income but also on past income saved and perceived future income. This conceptual framework is called permanent income, which, predictably, was developed in the Permanent Income Hypothesis. In its simplest form, permanent income is defined by the equation:

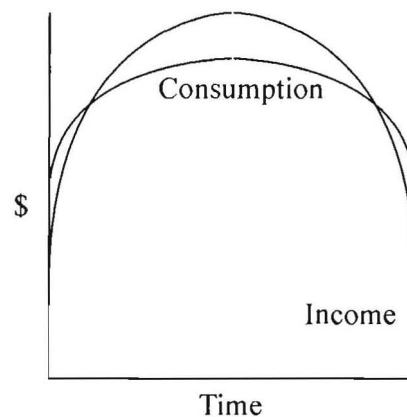
$$\text{Permanent Income (PI)} = \frac{\text{Total Lifetime Income}}{\text{Total Lifetime}} = \text{Annual Consumption (C)}$$

Let us take a close look at this equation. Because total income is divided evenly among each year of life, this equation implies that there is perfect smoothing in consumption. In other words, even though income may vary over the course of a lifetime (i.e. people usually earn lower incomes when they are very young and very old), consumption does not vary. While the idea of smoothed consumption is at least plausible, the idea that consumption is perfectly smoothed is probably erroneous. Perfect smoothing is depicted in Figure 2.1 below. Notice consumption does not change at all as income changes. Casual empirical observation contradicts the notion of perfect smoothing occurring in the real world, where agents generally increase consumption as income increases. This more realistic case is depicted in Figure 2.2. Note that consumption does rise as income rises, but there is still some smoothing that occurs. The important takeaway from this discussion is that permanent income hypothesis describes a consumption curve that is relatively flat in comparison to the income curve.

**Figure 2.1**



**Figure 2.2**



Aside from the plausibility of the occurrence of perfect smoothing, there are two additional problems that the permanent income equation has. The first of these is that *total*

*lifetime income* assumes that an agent knows how much income will be made during working years. The second problem is that *total lifetime* assumes that an agent knows how long he or she is going to live. While these assumptions are of course preposterous, they do not render the idea of permanent income completely inadequate for two reasons.

The first reason is that enough information is available for free from several sources about careers and health to reasonably estimate an annual income and an expected lifetime. For example, consider an 18-year old American female contemplating entering college to become a registered nurse. She knows that she will live about 80 years, and if she pursues a nursing degree she can expect an annual income of about \$60,000. Let us ignore interest rates, any inheritance she wants to leave her children, and the possibility of periods of unemployment. At the same time, we will assume that she will work for 45 years. Her permanent income is then calculated as such:

$$\text{Permanent Income} = \frac{\$60,000 \times 45 \text{ years}}{80 \text{ years} - 18 \text{ years}} \approx \$43,500$$

If she chooses not to attend college her salary would be in the neighborhood of \$30,000 (the average income of those with only a high school diploma). However, because she is not going to college she gains another 4 years of income. Her permanent income is then:

$$\text{Permanent Income} = \frac{\$30,000 \times 49 \text{ years}}{80 \text{ years} - 18 \text{ years}} \approx \$24,000$$

Thus, if she chooses to go to college she is raising her permanent income, and therefore her consumption, substantially. If she is only concerned with maximizing consumption, she should choose to go to college (of course, there are many other factors that weigh in on this decision).

The second reason that the idea behind permanent income is valid, despite our

oversimplification, is that while agents may not explicitly calculate permanent income as we did above, they may do so implicitly. In other words, even though agents might not go through the process of calculating permanent income, they might use common sense and logic to lead them to the same consumption decisions. For example, most students entering college probably do not determine how much debt to accumulate based on how much a college education will increase their permanent income. What they probably do determine is that a college education will mean they can earn a higher income. This in turn likely means they will be able to both pay off their school debt, and also consume more than they would have been able to had they not gone to college. The end result is that consumption decisions are the same whether permanent income is explicitly calculated or not.

Let us now examine some numerical examples to see what effects the Permanent Income Hypothesis and unexpected income changes (the actions of others) have on optimal consumption decisions. First, let us divide the life of an agent into three periods based on income (I). The early period (E) includes the higher education or the early career stages of life where little or no income is earned. For our purposes, let us suppose that income in this period is \$15,000. The middle period (M) includes the working years where income peaks and most of the total lifetime income is earned. Let us assume an income of \$60,000 in this period for our example. The late period (L) includes the years of retirement and old age where income is again low and agents live mostly off of savings. For simplicity, let us suppose that income during this period is \$15,000. Let us again ignore interest rates and inheritances. Let us also make the assumption that permanent income (PI) is after tax permanent income and that consumption in each year is equal to permanent income ( $C = PI$ ). Using our formula, let us quickly calculate permanent income,



shown below:

$$\text{Permanent Income} = \frac{\$15,000 + \$60,000 + \$15,000}{3} = \$30,000$$

Table 2.1 shows the base consumption decisions that will be used to compare consumption decisions in other situations. It displays the income and permanent income earned in each period.

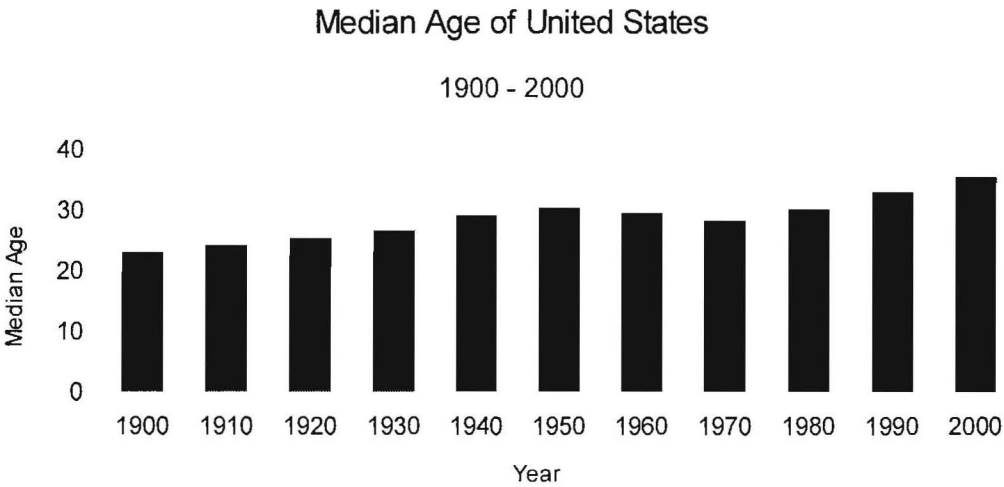
**Table 2.1**

Base	E	M	L
I	\$15,000	\$60,000	\$15,000
PI=C	\$30,000	\$30,000	\$30,000

It should be noted that, if the Permanent Income Hypothesis holds, consumption is much more even over lifetime than is income (in this case, consumption is perfectly smoothed). Also, agents who are in the early or late periods are spending more than they earn. That is, they depend either on credit markets or past saving to consume at their permanent income level. From this, we can conclude that the aggregate saving in an economy is at least partly dependent on the number of agents in their middle period. More generally, it can be said that demographics can have an important role in consumption and saving. For instance, countries that have young or aging populations (most people are in the (E) or (L) period) are most likely going to have lower rates of saving than do countries with a higher proportion of middle aged people. One way this can affect the economy is due to the fact that investment is related to savings. For a real world example, let us look at the United States. Figure 2.3 displays the median age of U.S. Citizens over time. Permanent Income Hypothesis would predict an aging population to have a falling savings rate.

Note that the median age has been slowly increasing over the last century, despite some fluctuations. Now consider Figure 2.4, which shows the average personal savings rate of the United States over time. Note that since about 1980, the savings rate has experienced a trend of significant decline. Referring back to Figure 2.3, the age of the United States has seen a relatively rapid increase since 1980. While these two facts alone do not prove that the Permanent Income Hypothesis is correct in stating that older populations cause lower savings rates, they do indicate a possible correlation.

**Figure 2.3**



*Source: U.S. Census Bureau, decennial census of population, 1900 to 2000.*

**Figure 2.4**



*Source: U.S. Department of Commerce: Bureau of Economic Analysis*

Now let us look at what happens to consumption decisions when income changes unexpectedly. These changes can take the form of either temporary changes or permanent changes. Let us consider an example of each. Suppose that in the early period, an agent wins the lottery and receives a lump sum of \$30,000. Other than this, the agent earns the same amount of income as in Table 2.1 during the middle and late periods. That is, there is a temporary increase in income in the early period. Let us now recalculate permanent income, taking this change into account. This is shown below:

$$\text{Permanent Income} = \frac{\$45,000 + \$60,000 + \$15,000}{3} = \$40,000$$

The effect on permanent income and consumption decisions is shown in Table 2.2 below.

**Table 2.2**

Temporary	E	M	L
I	\$45,000	\$60,000	\$15,000
PI=C	\$40,000	\$40,000	\$40,000

In terms of consumption decisions, the temporary increase of \$30,000 in income is divided equally among the three periods. In comparison to Table 2.1, consumption increases by 33 percent. More importantly, it should be observed that consumption in each period increases by less than the full \$30,000 amount of the lottery winnings.

Now suppose that the lottery is designed such that the agent receives \$30,000 in each period. With this change in mind, let us again calculate permanent income, shown below:

$$Permanent\ Income = \frac{\$45,000 + \$90,000 + \$45,000}{3} = \$60,000$$

The effect of this change is summarized in Table 2.3.

**Table 2.3**

Permanent	E	M	L
I	\$45,000	\$90,000	\$45,000
PI=C	\$60,000	\$60,000	\$60,000

In comparison to Table 2.1, we can see that consumption has increased by 100 percent. More noteworthy is the observation that consumption in each period has increased by the full \$30,000 payment received. By comparing Tables 2.2 and 2.3, we can see that, while income is the same in the early period (\$45,000), consumption is substantially higher (\$60,000 as opposed to \$40,000). From this we can conclude that the Permanent Income Hypothesis implies that permanent changes to income have a much greater impact on optimal consumption decisions than do temporary changes. This implication is of great importance to the discussion of the following sections.

### **3. An Introduction of Uncertainty and Application to Tax Policy:**

In the previous section, we related the decision making process to the Permanent Income Hypothesis in an attempt to describe how agents make their consumption decisions. We then explored ways in which agents can make these decisions both explicitly and implicitly. Consumption decisions were expanded to cover a longer time horizon (specifically one lifetime), and the section was concluded with a discussion of how changes in income affect consumption decisions in the remaining periods of an agent's life. The change in income was assumed to be caused by an agent winning the lottery. Recall that the method by which the lottery winnings were transferred to the agent had a significant impact on the changes in the agent's optimal consumption decisions. A lottery which utilized a one time payment scheme created only a temporary change in the agent's income, and it was distributed equally among the remaining periods in the agent's lifetime. A lottery which paid the winnings in each period created a permanent change to the agent's income in each period, and had a much larger effect on optimal consumption decisions during the remaining periods of the agent's life.

While the lottery examples may be illustrative in showing the mechanism behind consumption decisions formulated in the Permanent Income Hypothesis, there are two problems with the examples. The first problem is that the changes in the agent's income were due to the agent's luck in winning the lottery. This is too unrealistic if the idea is to be expanded and applied to the decisions of all agents that make up an economy. A new, more realistic impetus for change in income must be discovered in order to help validate the applicability of the theory laid out in the previous section. The second and most important problem is that the lottery examples implied that the agent knew with certainty how the lottery winnings were going to be transferred.

In fact, this was the entire basis for the difference in the agent's optimal consumption decisions. It is often the case that the real world is not so certain. This creates a need to incorporate uncertainty into the theory in order to make it more applicable. The first part of this section will be dedicated to solving the two problems just outlined. The incorporation of these solutions will lead us to another important concept, called time inconsistency. That concept will be the focus of the latter half of the section.

First, let us address the issue of realistic causes for changes in income. Significant lottery jackpots are simply not won by enough people to expand the ideas of permanent income and optimal consumption decisions to a national economy. Fortunately, there are many occurrences that can affect incomes. The challenge is to choose an occurrence that affects an entire economy simultaneously (i.e. it affects a large number of people at about the same time). To do this we take into consideration government taxation policies. Suppose that a recently elected public official believes that after tax incomes are too low and wants to lower taxes in order to put more money into public hands (the public being made up of a multitude of economic agents). There are two kinds of policies that the public official could adopt. One policy is to create a temporary tax break, while the other policy is to adopt a permanent tax break. Both policies constitute an increase in after tax income to a large amount of economic agents during one time period. This solves our first problem of determining a more realistic impetus for changes to income.

Next, let us address the issue of certainty. There are a multitude of ways to incorporate uncertainty into the agent's decision making process, but let us endeavor to do it in a relatively simple manner. This can be done by building incomplete information into the structure of the tax break policy. To do this, assume that when the public official is elected to office, it is given that a

tax break policy of one of the two forms mentioned above will be passed by the legislature. Let  $T$  denote the government selecting a temporary tax break policy and  $P$  denote the selection of a permanent tax break policy. Further assume that the policy will not be passed until late in the year (i.e. after the public has made its consumption decisions). Because of this feature, the public must form one of two expectations at the beginning of the year. First, the public can expect the tax break policy to be temporary, represented by  $t$ . Second, the public can expect the tax break policy to be permanent, represented by  $p$ .

Before we explore the outcomes that result from different combinations of tax break policies and public expectations, there is one important issue that needs to be addressed. Let us call it the issue of heterogeneity in human expectation. We have so far made an implicit assumption that all of the agents in the economy are forming the same expectations about the government's tax break policy, so that expectations are homogeneous. In reality, humans disagree with each other and form different opinions and expectations in nearly every aspect of life. This fact, however, does not destroy the validity of the implications for the scenario we have built. While all of the agents in the economy may not have the same expectation for tax break policy, there is a likelihood that a majority of agents, large or small, will have the same expectation. This only changes the magnitudes, rather than the characteristics, of the outcomes to our scenario.

Let us now discuss the outcomes that result from the different combinations of tax break policies and public expectations. There are four possible outcomes, summarized in the tables below. The letters in the top left cell of each table represent the tax break policy selected by the government and the expectation that the public has about the tax break policy. The letters on the left are either  $T$  or  $P$ , while the letters on the right are either  $t$  or  $p$ . Remember that the capitalized

letters represent the government's policy selection and the lower case letters represent the expectations of agents.

The first two tables should look familiar. Table 3.1 is similar the case where the lottery winnings were temporary. In this scenario, the government has chosen a temporary tax break policy, and the public expected the government to choose this policy. The resulting consumption decisions made by the agent are identical to those made in Table 2.2 of the previous section. Table 3.2 is similar to the case where the lottery winnings were paid in each period. In this scenario, the government has chosen a permanent tax break, and the public expected the government to choose this policy. The consumption decisions that result are identical to those of Table 2.3 in the previous section.

**Table 3.1**

$T, t$	E	M	L
I	\$45,000	\$60,000	\$15,000
PI=C	\$40,000	\$40,000	\$40,000

**Table 3.2**

$P, p$	E	M	L
I	\$45,000	\$90,000	\$45,000
PI=C	\$60,000	\$60,000	\$60,000

Table 3.3 and Table 3.4 are substantially different. In Table 3.3, the the government selected  $T$ , the temporary tax break policy, while the public selected  $p$ , expecting the government to employ the permanent tax break policy. The result of this discrepancy between reality and public expectations cause a large increase in consumption during the early period but no increase



to consumption in the middle and late periods. This occurs because the public acts as if the tax break policy is permanent in the early period, and consume all of the additional income from the tax break. We can model this decision by using the permanent income equation for expectations of a permanent tax break. This equation is:

$$Permanent\ Income = \frac{\$45,000 + \$90,000 + \$45,000}{3} = \$60,000$$

In the middle period, after the public recognizes that the tax break policy was temporary, it must adjust its expectations and recalculate permanent income for the middle and late periods. To do this, it must take into account the \$15,000 loan incurred during the early period by subtracting it from income earned in the middle and late periods. This new calculation is shown below:

$$Permanent\ Income = \frac{\$60,000 + \$15,000 - \$15,000}{2} = \$30,000$$

The process we have just completed is summarized in Table 3.3, which shows the income earned in each period, as well as the corresponding consumption decisions.

**Table 3.3**

<i>T, p</i>	E	M	L
I	\$45,000	\$60,000	\$15,000
PI=C	\$60,000	\$30,000	\$30,000

In Table 3.4, the government selected *P*, the permanent tax break policy, while the public selected *t*, indicating that expectations for the government to use the temporary tax break policy. The result of the divergence between reality and the public's expectations in this situation was a small increase in consumption during the early period and a relatively large increase in consumption during the middle and late periods. This occurs due to the fact that in the early

period, the agents act as if the increase to income is temporary, therefore distributing the gain equally amongst all three periods. In terms of a permanent income equation, this is represented by the following calculation:

$$Permanent\ Income = \frac{\$45,000 + \$60,000 + \$15,000}{3} = \$40,000$$

In the middle period, after the public recognizes that the tax break policy was permanent, it must adjust expectations and distribute the remaining permanent income to optimize consumption decisions in the middle and late periods. The permanent income calculation for this adjustment is shown below:

$$Permanent\ Income = \frac{\$90,000 + \$45,000 + \$5,000}{2} = \$70,000$$

Table 3.4 below summarizes what we have just discussed.

**Table 3.4**

<i>P, t</i>	E	M	L
I	\$45,000	\$90,000	\$45,000
PI=C	\$40,000	\$70,000	\$70,000

It should be noted that each scenario discussed in this section represents an extreme example, where the public was either completely correct in its expectations or completely wrong. However, it is completely possible (and probable) that agents select consumption levels which are in between the values depicted in the tables. In this respect, what the tables represent are boundaries. They demonstrate scenarios where expectations perfectly match reality and where expectations are perfectly contrary to reality. In practice, what actually occurs is located somewhere between the boundaries that we have established.

Two important conclusions can be drawn from the above examples. First of all, including uncertainty into the model of consumption decisions creates the potential for situations where consumption decisions are not optimal. These are situations where the agent does not evenly distribute permanent income between all three periods, and this happens due to uncertainty of the government selection of tax break policy. The second and more interesting conclusion is that it is not the tax break policy selected by the government that matters. It is the expectation of the agent that matters in determining consumption decisions. To illustrate this conclusion, let us take another look at the tax break policy example. Note that in both Table 3.1 and Table 3.4, the agent expects the government to pursue a temporary tax break policy, and makes the same consumption decision in the early period. The consumption decision is the same even though the government selected different tax break policies in the two situations. Similarly, in both Table 3.2 and Table 3.3, the agent expects a permanent tax break policy to be selected, and makes the same consumption decision in the early period for both situations. Again, different policies were actually selected by the government in the two scenarios. To reemphasize the importance of the conclusion, it should be again stated that what matters in determining consumption decisions is not the tax break policy the government selects but rather the expectation of which policy the government will select.

#### **4. An Introduction to Time Inconsistency and Application to Discretionary Policy:**

In the previous section, it was shown that when agents have incomplete information, their short run (i.e. early period) expectations may be incorrect, leading to suboptimal consumption decisions. In the middle period, the agents saw that the tax break policy was either temporary or permanent, and adjusted their expectations accordingly. In other words, while in the short run

(early period) the public expectations could be wrong, in the long run (middle and late periods) agents adjusted their expectations to be correct. This has important implications for the government's optimal choice of policy. Because agents change their expectations over time, governments often face a complex issue, which has been described as the problem of time inconsistency.

Before we begin to discuss an example of the time inconsistency problem, let us first create a better definition of what the problem is. Up to this point in our analysis we have only considered optimal decision making on the part of the public (made up of many agents). However, governments are also agents, and they have goals and agendas which they want to optimize on as well. One way governments do this is through the form of discretionary policy decisions (such as the tax break policy from the previous section). If the government's optimal decision for policy in the short run is the same as the optimal decision in the long run, then the long run policy is said to be time consistent. Conversely, if the government's optimal policy decision in the short run differs from that of the long run, then the long run policy is said to be time inconsistent (Romp 118). Let us now build an example to demonstrate the conditions that can lead to time inconsistent policy decisions.

Suppose the discretionary policy in question is the targeting of output (income) levels. Let us make the example fairly simple by assuming that the government has only two options for its policy decision. The government (represented by  $G$ ) can either choose a moderate policy, designated as  $M$ , or an aggressive policy, designated as  $A$ . Further assume the  $M$  is the equilibrium level of output and that  $A$  is a level of output above the equilibrium. If the government chooses an aggressive targeting policy, inflation will increase because output is

above the equilibrium level. In this scenario, the private sector (represented by PS), made up of agents, can choose between two expectations for the amount of inflation, where  $H$  denotes expectations of high inflation and  $L$  denotes expectations of low inflation.

With a framework of the scenario in place, we can now detail the progression of events. At the beginning of the period, the private sector must choose to expect either high inflation or low inflation. With the knowledge of what the private sector has chosen, the government is then free to choose a moderate or aggressive targeting policy. In this situation, the government and the private sector each have two possible choices, so there are four possible combinations of choices. Each combination of choices is associated with a payoff for both the government and the private sector. Let us make the assumption that agents are interested in maximizing their payoffs.

Now, let us determine the payoffs for both agents. Suppose that the private sector is only interested in whether or not the expectation it has matches the inflation actually experienced. If it is, the payoff is 1. If not, the payoff is 0. Note that when output is at the equilibrium, inflation will be as expected (Carlin 641). A few key assumptions must be made about the government's preferences before discussing its payoffs. If the expected inflation rate is low, then the government would prefer to choose a slightly higher inflation rate in order to pursue an aggressive targeting policy. However, when expected inflation is high, the government would prefer to keep inflation as low as possible by choosing a moderate policy (Carlin 641). The government's payoffs can now be formulated based on these preferences. If the private sector expects high inflation, then a moderate policy has a payoff of 2, while an aggressive policy has a payoff of 1. If the private sector expects a low inflation, then an aggressive policy has a payoff of 4, while a moderate policy has a payoff of 3.

Now it is time to put all of the information together into a visual chart, which will make it easier to derive insights from the four possible outcomes. Figure 4.1 displays the information below.

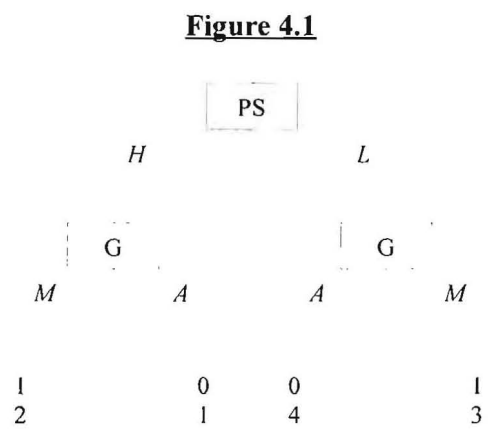


Figure 4.1 should be viewed in the following way. Each box is called a decision node, with the letters inside the node representing the agent whose turn it is to make a decision. The letters next to each line represent a decision, and the numbers at the bottom represent the payoffs for each agent (the top number corresponding to PS, the bottom to G). The progression begins with the top decision node, and PS chooses either *H* or *L*. Depending on the choice, we move down either the left or right line, at which point we reach a decision node for G. Then G makes its decision between *M* or *A*, and we move down the corresponding line to the associated payoffs. At this point, the progression has finished and the payoffs for each agent can be analyzed.

For the government *G*, there is an ideal progression which would allow it to maximize its payoff. This happens when the private sector *PS* chooses *L*. When this happens, the government will get a payoff of either 3 or 4, both of which are better payoffs than the government could receive should the private sector choose *H*. To insure this outcome, at the beginning of the period

the government may promise the private sector that it will choose a moderate policy ( $M$ ) if the private sector chooses  $L$ . Given that each group is trying to maximize its payoff, the private sector can give no credibility to the government's promise, because if the private sector chooses  $L$  then the government will choose  $A$  in order to maximize its payoff. This would leave the private sector with a less than optimal payoff itself. Faced with this knowledge, we must seek to find out the optimal decision for the private sector. If the private sector chooses  $H$ , then the government will choose  $M$  in order to maximize its payoff. The private sector will receive a payoff of 1 based on this action, thus maximizing its payoff. In light of this reasoning, we can conclude that the best choice for the private sector is  $H$ .

At this point, it may appear that the idea of time inconsistency has been lost, though in reality it has not. The problem that the government has is that it cannot credibly commit itself to a targeting policy until the private sector has set its expectations. This is because it cannot maximize its payoff if it holds to its commitment on targeting policy. The government's optimal policy in the short run (beginning of the period) is  $M$  because this could induce the private sector into choosing  $L$ , thereby ensuring the government's ability to maximize its payoff. Unfortunately, in order to achieve that maximum payoff, the government must deviate from its optimal short run policy of  $M$  and choose its long run optimal policy of  $A$ . The government's optimal long run policy differs from its optimal short run policy, which matches the definition of a time inconsistent policy developed near the beginning of this section. The reason for why rationally driven agents PS and G cannot coordinate their choices in order to achieve a more optimal joint outcome is not yet apparent. This issue is discussed in the following section.

## **5. An Introduction of Game Theory and Two Macroeconomic Games:**

The previous section was concluded with a brief discussion of how both the private sector and the government made optimal decisions given our assumed conditions, and why the government's optimal policies were time inconsistent. In this section, we will begin by introducing game theory into our time inconsistency situation. Next, certain concepts of game theory will be examined more closely. Finally, the section will end with the discussion of a new game to introduce more game theory topics and applications.

Let us begin by defining what exactly a game is. There are three conditions that are necessary to have a game. First, a game must consist a set of two or more players (or agents). Second, each player must have a set of moves (or choices) available to them. Finally, there must be a specification of payoffs to each player for each combination of moves (Fudenberg 4). There are several ways in which games can be represented, but for our purposes we will use extensive form because it is more intuitive and yields the same end results after analysis. In addition to the three conditions of games described above, extensive form games display three additional pieces of information. The first is the order that players move (i.e. who moves when). The second is the information that each player has available when making a choice. The third is a probability distribution of any exogenous events (Fudenberg 77). We will begin our discussion of game theory by re-examining the game between the private sector and the government from the previous section.

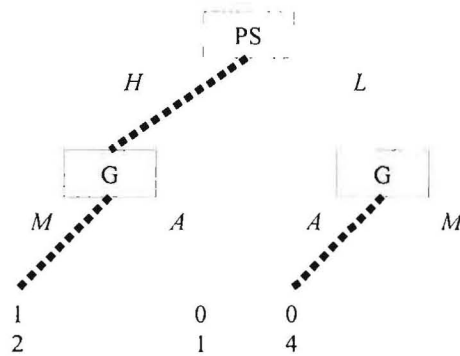
Figure 5.1 is essentially identical to Figure 4.1, but we will now use game theory to “solve” the game by using a method called backward induction. This method is applicable to extensive-form games that have a defined number of moves. We begin by considering the



decision nodes where, after the player chooses a move, the game ends. These are called terminal decision nodes, and there are two of them in Figure 5.1. These are the two G decision nodes. For each of these terminal decision nodes, the optimal choice for the player at that point is evident because we can see the payoffs associated with each choice. For example, at the left hand decision node G, the government chooses *M* because its payoff is 2, whereas the payoff for choosing *A* is only 1. The optimal choice for each terminal decision node is characterized by a thick dashed line, which can be seen in Figure 5.1. The same process is repeated for all remaining terminal decision nodes (in our case there is only one more, the right hand G decision node).

Having done this for all terminal decision nodes, we then move up the game tree to the next level of decision nodes. In our case, this is the PS decision node, and it is the only node at this level. The payoff associated with each choice for PS can be seen because we have already determined what G will do for either of the choices PS can make. The payoff to PS from choosing *L* is 0, because if *L* is selected G will choose *A*. Using similar rationale it can be seen that the payoff to PS from choosing *H* is 1. Thus, *H* is the optimal choice for PS, so we make the line associated with that choice a thick dashed line. At this point, there are no further decision nodes to consider; the game has been solved. The solution moves down the thick dashed lines from the start of the game until it ends at the payoffs. This path is called the equilibrium path solution (EPS) of the game (Friedman 47).

**Figure 5.1**



At this point, game theory has not given us any additional information, as we were able to find the solution to the game in the previous section. Let us now address this issue by moving on to the next portion of this section. If we examine the payoffs in Figure 5.1, it can be seen that the outcome that maximizes the joint utility of both agents is not the equilibrium path outcome. This occurs when PS chooses  $L$  and  $G$  chooses  $M$ , with payoffs of 1 and 3, respectively. Comparing this outcome to the EPS, PS is just as well off and  $G$  is better off. Game theory explains exactly why it is rational for a less than optimal outcome to be the EPS using the concept of a sub-game perfect equilibrium (SGPE).

First, let us define the sub-game. It is simply a game which starts at a decision node (Friedman 43). There are three sub-games in Figure 5.1; in fact the entire game itself can be classified as a sub-game. The idea of an SGPE is that an agent behaves rationally at each of that agent's decision nodes. This rational decision making at each node is called a strategy. A strategy will be denoted by brackets  $\{ \}$ . PS has only one decision node and thus only has two strategies of  $\{e = H\}$  and  $\{e = L\}$  ( $e$  represents PS expectation).  $G$  has two decision nodes, and it has strategies of the following form:  $\{\text{if } e = H, \text{ then } c = M; \text{ if } e = L, \text{ then } c = A\}$ , where  $c$  represents

the policy choice that G makes. What defines an SGPE is that, for all sub-games in a game, the agent that must make a choice at the beginning of the sub-game must have a rational strategy from that point forward, given the other agent's chosen strategy (Carlin 644). Our game is relatively simple in that the two sub-games which start at G decision nodes, no further strategies need be taken into account since they are terminal decision nodes. At the sub-game that starts at the PS decision node, PS only has to consider G's strategy from that point forward. These considerations are represented by the thick dashed lines in Figure 5.1. Because of their nature, there is a guaranteed SGPE for games of this form (Carlin 644).

To summarize, what we have discovered so far is that solving finite games with backward induction implies that the solution is an SGPE. This means that non-credible commitments or threats can be identified, and game theory has proven itself very useful in determining why time-inconsistency problems occur between agents that are acting rationally. In the interest of applicability to real world situations, there is still one problem we must address. In our time-inconsistency game, both players had complete information, meaning that the payoffs and strategies of each agent were known to both agents. Games of this sort certainly do occur, but games of incomplete information are typically more interesting and more important in economic applications. Let us turn our attention to games of incomplete information presently.

Consider the possibility that in order to reduce the problem of time-inconsistency, the government delegates authority for monetary policy to an institution with stronger incentives to fight inflation. This institution usually takes the form of a central bank (CB). This may sound like a good idea, but of course, the central bank may be a farce and simply act as a government puppet. When this is the case, the bank follows the same kind of policy decisions made by the

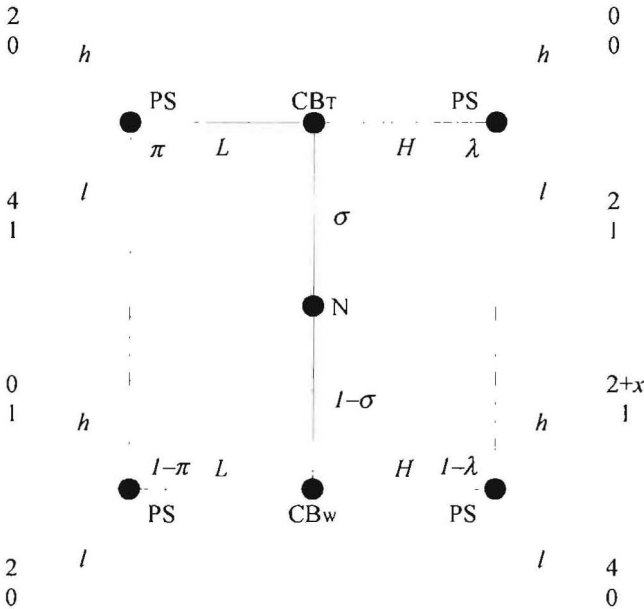
government in the preceding example. The key difference, however, is that the private sector would not initially know which kind of central bank it is dealing with. This creates a kind of game where one agent lacks information about the preferences of the other agent. This presents the latter agent with the opportunity of sending a signal (either truthful or misleading) to the former indicating what those preferences are. Let us now turn to an example where there is uncertainty for the private sector about the central bank's preferences.

Let us first outline the game. There are three moves in this game, presented in Figure 5.2. Decision nodes are represented by large dots, and the game begins at the central dot, where nature (N) determines the type of CB, moving the game to either decision node  $CB_T$  or  $CB_W$  with probabilities of  $\sigma$  or  $1-\sigma$ , respectively. This determination is not observed by PS, who only knows the respective probabilities of the determination. Next, CB moves by either signaling high ( $H$ ) or low ( $L$ ) level of output. This move is observed by PS, but PS does not know whether the move was made by  $CB_T$  or  $CB_W$ .

Suppose that  $L$  was signaled. If the type of CB was  $CB_T$ , then the game moves from the  $CB_T$  decision node to the top left PS decision node. If the type of CB was  $CB_W$ , then the game moves from the  $CB_W$  decision node to the bottom left PS decision node. Thus, PS knows that it is at one of the two decision nodes on the left, but it does not know which particular node it is at because it does not know the type of CB it is dealing with. When this situation occurs in a game, the decision nodes involved are called an information set (Carlin 653). Information sets are shown as dashed lines connecting relevant decision nodes in Figure 5.2. In games, when it is an agent's turn to move, the agent always knows which information set it is at, but does not necessarily know which particular decision node it is at. In our example, we supposed  $L$  was

signaled, so PS knows that it is at the left hand information set but does not know if it is at the top or bottom decision node. At this point, PS must make its move, choosing to expect either high ( $h$ ) or low( $l$ ) inflation. The game then ends with the payoffs, the top number representing the payoff to CB and the bottom number to PS.

**Figure 5.2**



Let us now take a moment to discuss the rationale behind the payoff structure, beginning with payoffs to PS. Remember that the choice of  $H$  or  $L$  by CB is only a signal to PS of the policy it will choose, not its actual choice of policy. We assume that the tough central bank,  $CB_T$ , will always choose  $L$  regardless of the signal it sends to PS. Thus, if  $CB_T$  signals  $L$  and PS chooses  $l$ , expectations will match inflation levels and PS gets a payoff of 1. If  $CB_T$  signals  $L$  and PS chooses  $h$ , expectations will not match inflation levels and PS gets a payoff of 0. The weak central bank,  $CB_W$ , will always pursue  $H$  regardless of its signal to PS. Thus, if  $CB_W$  signals  $L$

and PS chooses  $h$ , then expectations will match inflation and PS gets a payoff of 1. Conversely, if  $CB_w$  signals  $L$  and PS chooses  $l$ , expectations will not match match inflation and PS gets a payoff of 0. Similar rational can be used to analyze the PS payoffs of the right information set.

Now let us turn to payoffs for CB. The tough bank,  $CB_T$ , prefers to signal  $L$  and prefers PS to choose  $l$ . Thus, it receives its highest payoff of 4 when this occurs. If only one of its preferences are met (either it signals  $L$  or PS chooses  $l$ ), then the payoff will be 2. If neither of its preferences are met (it signals  $H$  and PS chooses  $h$ ), it receives its lowest payoff of 0. The weak bank,  $CB_w$ , prefers to signal  $H$  and prefers PS to choose  $l$ . It receives its highest payoff of 4 when this happens. When only one of these preferences are met, it will receive a payoff of 2, and receives a payoff of 0 if neither preference is met.

Before we begin to analyze this game in detail, one further point needs to be made. Just because PS does not know which particular decision node it is at in a given information set, this does not inhibit it from making a sensible guess. In fact, in this type of game, it is a crucial element that PS does its best to determine the probability of being at one decision node as opposed to the other. Let us use  $\lambda$  to represent the probability that PS assigns to being at the top decision node in the right information set, making  $1-\lambda$  be the probability of being at the bottom decision node. Similarly,  $\pi$  will represent the probability of being at the top decision node in the left information set,  $1-\pi$  being the probability of being at the bottom decision node.

When looking for a possible equilibrium in a signaling game, we must leave the concept of the SGPE behind. This is because it can only fully solve games in the absence of uncertainty about which decision node an agent is at. This is due to the fact SGPE checks rational behavior of an agent if their move begins a sub-game, which only start from single decision nodes.

Therefore, the concept of SGPE is not very useful for games containing information sets, but the basic idea behind them can be altered for use in such games. This is called a Perfect Bayesian Equilibrium (PBE), which accounts for the uncertainty contained in an information set. A PBE essentially has three conditions. The first is that at each information set, the agent whose turn it is to move must assign probabilities to the decision nodes within the set, and these probabilities are assumed to be known by all agents. Second, given these probabilities, the agent's strategy from then on must take into account the strategies of other players from that point forward. Finally, the probabilities must be chosen as rationally as possible given the strategies of the agents. Let us apply the concept of PBE to our game in order to see how it works.

We begin by assigning probabilities within the information sets. There are two sets, one corresponding to a CB signal of  $H$  and the other corresponding to a signal of  $L$ . Let us consider information set  $L$ . Again, all PS knows is that  $L$  has been signaled, and that there is a  $\pi$  chance that CB is  $CB_T$  and a  $1-\pi$  chance that CB is  $CB_w$ . Now let us consider the strategies given these probability assignments. When signal  $L$  is received, the expected payoff to PS if it chooses  $h$  is:

$$\pi * 0 + (1 - \pi) * 1 = (1 - \pi)$$

and the expected payoff to PS if it chooses  $l$  is:

$$\pi * 1 + (1 - \pi) * 0 = \pi$$

This means that PS should choose  $h$  if  $\pi < .5$  and choose  $l$  if  $\pi > .5$ . This should have some intuitive appeal. It means that PS will choose  $l$  if the likelihood of the central bank being tough is relatively high compared to the likelihood of it being weak.

Now let us look at things from the perspective of the central bank. Let us start by considering the tough bank  $CB_T$ , specifically if it pays to choose  $L$ . If  $\pi > .5$ , it is clearly a good

strategy to pick  $L$ , because then PS will choose  $l$  and the payoff to CB will be 4. Should  $CB_T$  choose  $H$ , the best possible payoff that CB could receive is 2, regardless of the choice PS makes. Thus  $CB_T$  is clearly better off choosing  $L$ . Now let us consider the weak central bank,  $CB_W$ . If  $CB_W$  chooses  $L$  and if  $\pi > .5$ , then PS would choose  $l$  and the payoff would be 2. Suppose  $CB_W$  chooses to signal  $H$ . The payoff that it receives is now dependent on  $\lambda$ , which we can determine using the assumption that PS behaves rationally. PS can reason that  $CB_T$  can get at best a payoff of 4 and at worst a payoff of 2 by choosing  $L$ , whereas it receives a payoff of 2 at best and 0 at worst by choosing  $H$ . This implies that it would never be rational for  $CB_T$  to signal  $H$ , and therefore PS should assign a probability of 0 to  $\lambda$ . Thus, if  $H$  is signaled, PS can infer that the central bank is  $CB_W$  and therefore would choose  $h$ . This eliminates the possibility of  $CB_W$  receiving a payoff of 4, which makes the value of  $x$  critical.

If  $x < 0$ , then  $CB_W$  will be better off choosing  $L$ , following the same strategy that  $CB_T$  follows. When this is the case,  $L$  is called a pooling equilibrium (both types of banks are pooling on the same strategy). If  $x > 0$ , then  $CB_W$  will be better off choosing to signal  $H$ , following the opposite strategy that  $CB_T$  follows. In this case, the difference in strategy is called a separating equilibrium (the signal selected separates the two types of banks). Finally, if  $x = 0$ , then  $CB_W$  will be indifferent as to which signal it selects. What we have discovered is that in situations of uncertainty, expectations (and the probabilities associated with them) have a strong impact on optimal policy choices. Additionally, we have seen that signals which agents send to one another have an important impact because they can influence these expectations. It is ultimately incentive structures (payoffs) that determine the significance of signals and how they are used by agents.



## **Conclusion:**

The goal of this paper has been to establish the prevalence of decision making in our lives, the processes we go through in order to make optimal decisions, and the implications of these decisions in the real world. We have seen simple decisions such as those made by players in a tennis match. We have also seen complex decisions such as the private sector making expectations for inflation based on incomplete information about the nature of a central bank. The theories and decision making situations discussed have often had significant impacts on real world phenomena, as we saw with the savings rate in the United States being affected by the age structure of the country and the resulting consumption decisions. Hopefully by carefully examining the way in which we make decisions, we can come up with ways to work around problems and make even better decisions. While this paper has not been extremely extensive or exhaustive on the topic of decision making, it does provide an introduction to relevant ideas which can be studied in more detail to gain further knowledge and understanding.

### **Works Cited:**

Carlin, Wendy, and David Soskice. *Macroeconomics: Imperfections, Institutions, and Policies*. Oxford: Oxford University Press, 2006.

Friedman, James W. *Game Theory with Applications to Economics*. Oxford: Oxford University Press, 1990.

Friedman, Milton. *A Theory of the Consumption Function*. Princeton: Princeton University Press, 1957.

Fudenberg, Drew, & Jean Tirole. *Game Theory*. Cambridge: The MIT Press, 1991.

Modigliani Franco, and Richard Brumberg. "Utility Analysis and the Consumption Function: An Interpretation of Cross –Section Data, in Post Keynesian Economics" ed by K. Kurihara. pp 388-436, Rutgers University Press, New Brunswick N.J, 1954.

Romp, Graham. *Game Theory: Introduction and Applications*. Oxford: Oxford University Press, 1997.

**Appendix. Another Application of the Permanent Income Hypothesis:**

Let us take a look at what happens when there is an unexpected decrease in income near the end of the middle period of life. At this point, savings are likely being built up in preparation for retirement. Suppose that each period in the base case (Table 2.1) consists of 30 years for a total lifetime of 90 years. Further, let us assume that at the end of the fifteenth year of the middle period, an increase in taxes diminishes current income and the ability to accrue savings. The result is that income (I) in the last 15 years of (M) and all 30 years of (L) is decreased by \$15. This is summarized in Table A.1.

**Table A.1**

Base Case	E	M	L
I	\$75	\$150   \$135	\$60
PI=C	\$100	\$100   \$85	\$85

Table A.1 shows that permanent income is also decreased by \$15 in the latter part of the middle period and during the late period. The agent at this point has two options. First, the agent can accept the decrease in permanent income and adjust to less consumption and therefore a lower standard of living. Second, the agent could choose to work longer and thus maintain the original permanent income of \$100. The agent’s problem is then deciding how much longer it is necessary to work. Total lifetime left is 45 years, so in order to have a permanent income of \$100, remaining lifetime income must be \$4,500, as shown by the permanent income equation.

$$\frac{\$4,500}{45\text{ years}} = \$100\text{ per year}$$

Now, using this information along with some algebra, the number of additional working years can be calculated. This process is shown below.

I.) First, it is known that income in the second half of the middle period and income in the late period must add up to \$4,500. This can be represented by the equation:

$$135M + 60L = 4,500 \quad (1)$$

II.) Second, it is known that the number of middle period years and late period years must add up to 45. This can be represented by the equation:

$$M + L = 45 \quad (2)$$

III.) Equations (1) and (2) form what is called a system of equations, shown below:

$$\begin{array}{rcl} 135M + 60L & = & 4,500 \\ M + L & = & 45 \end{array} \quad (3)$$

IV.) This system of equations can be solved in a number of ways. One of the most straightforward ways to solve the system is called elimination. The first step is to multiply one of the equations in the system by a number that will make the coefficient in front of one of the terms the same in both equations. Let us do this by multiplying each term in the second line of (3) by 60 to get the following :

$$\begin{array}{rcl} 135M + 60L & = & 4,500 \\ 60M + 60L & = & 2,700 \end{array} \quad (4)$$

V.) The next step is to eliminate one of the terms by addition or subtraction. In our system of equations (4), this can be done by subtracting the second line from the first line as shown below:

$$\begin{array}{rcl} 135M + 60L & = & 4,500 \\ -60M - 60L & = & -2,700 \\ \hline 75M & = & 1,800 \end{array} \quad (5)$$

VI.) We can now solve for  $M$  by dividing the last line in (5) by 75 to get the result:

$$M=24 \quad (6)$$

VII.) Finally, we can substitute (6) into (2) to obtain L as shown below:

$$24 + L = 45 \rightarrow L = 21 \quad (7)$$

What we have discovered is that the agent must now work for 24 more years as opposed to 15 more years before the income change. In other words, the late period has been shortened from 30 years to 21 years in order to main permanent income and consumption levels. Whether agents go through this process explicitly or implicitly, Permanent Income Hypothesis suggests that negative shocks to income create an incentive for agents to work longer than originally intended in order to maintain a standard of living.